[4] L. Dorst, D. Fontijne and S. Mann: *Geometric algebra for computer science: An object-oriented approach to geometry.* Elsevier/Morgan Kaufman, 2007. Revised edition 2009.
[5] C. Perwass: *Geometric algebra with applications in engineering.* Springer, 2009.
[6] L. Dorst and J. Lasenby (eds.): *Guide to geometric algebra in practice.* Springer, 2011.

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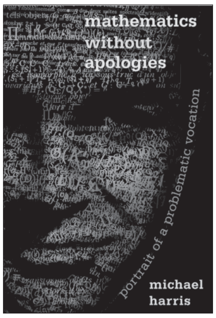

Michael Harris

Mathematics Without Apologies
Portrait of a Problematic Vocation

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Reviewer: Javier Fresán

What drives mathematicians? Why do some of the best minds, generation after generation, leave for seas of thought so far from their "first and authentic" lives? Standard answers to these questions are based upon on three apologies: mathematics is *good* because, regardless of how abstract a theorem seems to be today, it might well have unexpected applications in the future; mathematics is *true*, as it provides "timeless certainty" in a fast-moving world; and it is *beautiful*, although this art form is often hidden to the untrained eye. Avoiding clichés, or rather delving more into them, *Mathematics without apologies* argues that these may be motivations for the romanticised Mathematician but they are quite absent from the everyday life of working (small-m) mathematicians. Which takes us back to the initial question. In this playful, erudite, iconoclast essay, Michael Harris points to a few alternative solutions, including the sense of belonging to "a coherent and meaningful *tradition*", the participation in a *relaxed field* "not subject to the pressures of material gain and productivity" and the pursuit of a certain kind of pleasure. Its main merit, however, lies less in offering new answers than in seriously asking the right questions, perhaps for the first time. Inevitably, some readers will find the result irritating, a mere exercise of quotation dropping, while others will see a genuine piece of cultural criticism which reaches, at its best, the level of Bourdieu's *La distinction* or Foucault's commentary of *Las meninas*.

The book is divided into 10 chapters, together with a series of interludes around the easier question of "How to explain Number Theory at a Dinner Party". Here, Harris introduces the necessary background to state Hasse's bound for the number of points of elliptic curves over finite fields – the inspiration for Weil's use of trace formulas which "converted" the author to number theory – and give a rough idea of the Birch-Swinnerton-Dyer conjecture, which served as a "guiding problem" of his early career. Short explanations about prime numbers, congruences and polynomial equations are followed by an amusing, highly unlikely dialogue between two characters, a Performing Artist and a Number Theorist, who tease each other with quotations from Aristotle, Kronecker, Musil, Levinas and Stoppard. Had *Mathematics without apologies* consisted only of these pages, it would have already been an original work of popular science, with clever findings like the Galois group of Chekhov's *Three Sisters*. But they are simply intended as a complement to an inquiry of much larger scope, which can be skipped without detriment to the reader.

Far from seeing his discipline as a closed paradise to non-experts, Harris defends the fact that outsiders have contributed with valuable insights into what it means to live as a mathematician. What makes them especially relevant for his purposes is that they couldn't help but be conditioned by the public self-image that mathematicians project. A recurring theme of the book is how intentions are misrepresented, starting from the autobiographical writings of mathematicians themselves. For instance, most accounts of the vocation's awakening seem to overestimate the quest of certainty as a driving force. A typical example of how this and other commonplaces notions are turned around is the beginning of Chapter 2: "How I acquired charisma". In a breathless prose, Harris explains that his "mathematical socialisation" began the year when the Prague Spring, the May 1968 events and the riots after the assassination of Martin Luther King shook the foundations of the world he had known before. Luckily, he writes, mathematics was there "to take their place". However, if one continues reading the footnote afterwards, it becomes clear that it didn't really happen that way.

Chapter 6: "Further investigations of the mind-body problem" takes a closer look at how mathematicians are perceived; it is not by chance that the first and the last sentence contain the word "mirror". Some features of the reflected image, like absent-mindedness, persist through the centuries, whereas others have dramatically changed. Following science historian Amir Alexander, Harris illustrates the latter by contrasting the Enlightenment ideal of the mathematician as a "natural man" – exemplified by the encyclopaedist d'Alembert or the fictional geometer of Potocki's novel *The Manuscript found in Saragossa* – with the romantic archetype of the lonely, self-destructive hero, largely inspired by the myths surrounding Galois' death. If the young Stendhal could still think of mathematics as "the royal road to Paris, glory, high society [and] women", a few years later, mathematicians would not be considered good lovers anymore and painters would portray them as figures "absorbed by [their] own inner flame", with characteristic gleaming eyes, while physicists would keep their reputation of "successful men of the world". The chapter also links Edward Frenkel's film *Rites of Love and Math* to the historical search for a love formula and briefly evokes Hypathia's martyrdom and some cases of madness, to conclude that "our readiness to sacrifice our minds and bodies to our vocation is the ultimate proof that what we are doing is important".

A form of melancholy distinct from the sentimentality of the romantic mathematician traverses Chapter 7: "The habit of clinging to an ultimate ground", which addresses the fascinating question: "How can we talk to one another, or to ourselves, about the mathematics we were born too soon to understand?" The first paragraph describes the vertigo that Grothendieck's dream of a category of motives or the Langlands programme may cause. A quotation from André Weil, "one achieves knowledge and indifference at the same time", serves as a leitmotiv. In the article from which it comes, he explains that 18th century French mathematicians used to employ the word "metaphysics" to refer to vague, hard-to-grasp analogies, which nevertheless played an important role in mathematical creation. The term is nowadays replaced by "yoga" or "avatar", as well as a distinctive use of quotation marks or the word "morally" as an explicit "invitation to relax one's critical sense". Harris also calls attention to the peculiar use of the verb "exist" in mathematics and to the problem of understanding *uniqueness* once *existence* has been established. This leads him to discuss higher categories and Voevodsky's Univalent Foundations, in a train of thought which is at times challenging to follow.

Chapter 8 (one of the most interesting chapters) examines the role of tricks in mathematical practice. The goal is to decide what makes Cantor's diagonalisation trick or Weyl's unitarian trick (to cite a few examples) "trick[s] rather than some other kind of mathematical gesture[s]". In an illuminating archaeology, Harris tracks the first occurrences of the word in scientific writing and exposes the nuances of its translation into various languages. The matter is not so simple because different mathematicians give different meanings to trickiness; otherwise, how could one reconcile Grothendieck's dismissal of Deligne's solution to the last of the Weil conjectures because "the proof used a trick" with the commonly held view that it is one of the most outstanding results of the twentieth century? Harris enriches the classical dichotomy between theory builders and problem solvers with the figures of the *strategist* and the *technician*, who, in contrast to mythology, do not represent a functional division of labour but coexist in most individual mathematicians. The trickster may be seen "as a *bridge* between *high* and *low* genres", which brings the author to the question of why mathematics is systematically classed as a high genre. Or, to put it succinctly: "Why so serious?" The tentative answers form Harris' *Appocalitici e integrati.*

In Chapter 9: "A mathematical dream and its interpretation", the author narrates a dream about the cohomology of unramified coverings of Drinfel'd upper-half spaces, which changed his life "in more ways than [he] care[s] to name". It is hard to imagine what the general reader gets out of the actual content of the dream but I truly enjoyed this personal side-note to a literature where human aspects of creation tend to be reduced to their minimum expression. In most scientific dreams, the unconscious comes to the rescue only after the dreamer has relentlessly tried to solve a problem; this was the case of Thomason – as analysed in another brilliant essay by Harris[1] – who had worked for three years on the extension problem for perfect complexes before the simulacrum of his deceased friend Trobaugh offered the key to the solution. On the contrary, the author's dream sketches a strategy for making progress on a subject "to which [he] had devoted no passion". The question then arises of how those new ideas found their way into his dreams. Harris' interpretation, which is remarkably sincere, exemplifies another aspect of the mathematical pathos: competitiveness, which could be summarised in a quotation from a hedge-fund manager who quit research: "It is hard to do mathematics and not care about what your standing is."

Until now, this review has concentrated on the second and third parts of Mathematics without apologies. The first chapters raise a fundamental question: if mathematics cannot be justified as useful, true and beautiful then how can it be justified, especially when it comes to ask for funding? Harris criticises the use of mathematics to legitimise certain economic policy decisions, as well as other Faustian bargains related to the financial crisis; he also notices an increasing role of private philanthropy in mathematical research, which could end up jeopardising "the professional autonomy to which we have grown attached". A more interesting challenge to this autonomy, although less immediate, is the "paradigm shift" that an extended use of computer-assisted proofs and automatic proof-checkers could introduce. Despite his love for science fiction, Harris does not seem to worry too much about the possibility of machines substituting human mathematicians, as he understands that "the goal of mathematics is to convert rigorous proofs to heuristics". But we readers have got nothing to lose in adding a new question: will apologies still be needed in post-human mathematics?

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[1] M. Harris, "Do androids prove theorems in their sleep?". In A. Doxiadis and B. Mazur (eds.), *Circles Disturbed. The interplay of mathematics and narrative*. Princeton: Princeton University Press, 2012, pp. 130–182.