MOTIVES FOR PERIODS
AUGUST 28–SEPTEMBER 1, 2017

Minicourses

JOSEPH AYOUB – *Triangulated categories of motives and the Kontsevich-Zagier conjecture*

I will recall the construction of the triangulated categories of motives and discuss various related topics (the Betti and de Rham realisations, the rigid analytic variant, nearby motives, etc.). Then, I will recall the construction of the motivic Galois group and the torsor of motivic periods, and formulate the Kontsevich-Zagier conjecture on periods in this setting. Finally, I will formulate a geometric version of the Kontsevich-Zagier conjecture and explain its proof.

CLÉMENT DUPONT – *Mixed Tate motives and multiple zeta values*

Multiple zeta values (MZVs) generalize the values of the Riemann zeta function at integer points and form a fascinating algebra of real numbers. They appear in a wide variety of contexts, ranging from the theory of associators to the computation of amplitudes in particle physics. Since MZVs are periods, it is natural to introduce their motivic versions, which are acted upon by a motivic Galois group. Surprisingly enough, the Galois theory of motivic MZVs can be made entirely explicit and used to prove powerful theorems on real MZVs. The goal of this minicourse will be to explain the proofs of these theorems, with a special emphasis on Brown's recent proof of a conjecture of Hoffman. The relevant motivic framework is that of mixed Tate motives and their tannakian formalism, which we will review.

PETER JOSSEN – *Exponential motives and exponential periods*

In my lectures, I will present joint work with Javier Fresán. Our departing point is the observation that several interesting transcendence theorems and conjectures are about numbers which presumably are not periods in the usual sense, so Grothendieck's period conjecture says nothing about them. One such case is the Lindemann-Weierstrass theorem which implies for example that e and $e^{\sqrt{2}}$ are algebraically independent, another one is the Rohrlich-Lang conjecture which claims that for any integer $n \geq 3$, the transcendence degree of the field

$$\mathbb{Q}(\Gamma(\tfrac{1}{n}),\Gamma(\tfrac{2}{n}),\Gamma(\tfrac{3}{n}),\ldots,\Gamma(\tfrac{n-1}{n})) \qquad \text{with} \quad \Gamma(s) := \int_0^\infty x^{s-1}e^{-x}\,dx$$

is equal to $\frac{1}{2}\varphi(n)+1$. Another rich source of transcendence statements is the Siegel-Shidlovskii theorem, which shows for example that the number

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot n!} = \iint_{0 \leq x, y \leq 1} e^{-xy} dx dy$$

is transcendental.

In their celebrated paper on periods, Kontsevich and Zagier mention that it should be possible to enlarge Nori's tannakian category of mixed motives to a tannakian category of *exponential motives*, together with realisation functors and comparison isomorphisms between them. Whereas classical motives are associated to varieties, exponential motives are associated to pairs (X, f), where X is an algebraic variety, and f is a regular function on X. Periods for exponential motives, which we call *exponential periods*, typically look like

$$\int_{\gamma} \omega e^{-f}$$

where γ is a topological cycle on $X(\mathbb{C})$ and ω an algebraic differential form. In particular, all the examples above concern exponential periods, and can be recast in terms of the period conjecture extended to exponential motives.

An optimistic outline for my lectures:

(1) Construct some elementary cohomology theories for pairs (X, f), and then the category of exponential motives as a universal cohomology theory following Nori's method.
(2) Formulate the exponential period conjecture and give some examples and consequences. Compare to the classical period conjecture.
(3) Construct some more involved cohomology theories, in particular the Hodge realisation for exponential motives.
(4) Show on concrete examples how the Hodge realisation helps to compute motivic fundamental groups.

Talks

ISHAI DAN-COHEN – *Progress on rational motivic path spaces*

A central ingredient in Kim's work on integral points of hyperbolic curves is the "unipotent Kummer map" which goes from integral points to certain torsors for the prounipotent completion of the fundamental group, and which, roughly speaking, sends an integral point to the torsor of homotopy classes of paths connecting it to a fixed base-point. In joint work with Tomer Schlank, we introduce a space Ω of "rational motivic loops", and we construct a double factorization of the unipotent Kummer map which may be summarized schematically as

$$\text{points} \to \text{rational motivic points} \to \Omega\text{-torsors} \to \pi_1\text{-torsors}.$$

Our "connectedness theorem" says that any two motivic points are connected by a non-empty torsor. Our "concentration theorem" says that for an affine curve,

Ω is actually equal to π_1. As a corollary, we obtain a factorization of Kim's conjecture into a union of smaller conjectures with a homotopical flavor. With some luck, I'll also be ready to discuss the problem of delooping in this setting.

MARTIN GALLAUER – *Motivic Galois groups in characteristic 0*

I will survey different approaches by various mathematicians to constructing the Galois group for mixed motives over a field of characteristic 0. I will also try to elucidate the relation among these candidates, and explain why everyone interested in periods should care.

TIAGO JARDIM DA FONSECA – *Higher Ramanujan equations and periods of abelian varieties*

The Ramanujan equations are certain algebraic differential equations satisfied by the classical Eisenstein series E_2, E_4, E_6. These equations play a pivotal role in the proof of Nesterenko's celebrated theorem on the algebraic independence of values of Eisenstein series, which gives in particular a lower bound on the transcendence degree of fields of periods of elliptic curves. Motivated by the problem of extending the methods of Nesterenko to other settings, we shall explain how to generalize Ramanujan's equations to higher dimensions via a geometric approach, and how the values of a particular solution of these equations relate with periods of abelian varieties.

NILS MATTHES – *Twisted elliptic multiple zeta values*

We introduce an analog of multiple zeta values, which is naturally associated to an elliptic curve together with a distinguished set of torsion points, the so-called "twisted elliptic multiple zeta values". They generalize elliptic multiple zeta values, which were previously introduced by Brown–Levin and Enriquez, and are closely related to both cyclotomic multiple zeta values and iterated integrals of modular forms for congruence subgroups. In a similar way as mixed Tate motives over Z help to explain structural properties of multiple zeta values, it is hoped that the algebraic structure of twisted elliptic multiple zeta values can likewise be elucidated by a suitable category of mixed (elliptic) motives. This is joint work (partly in progress) with J. Broedel, M. Gonzalez, G. Richter, O. Schlotterer and F. Zerbini.

ERIK PANZER – *The Galois coaction on ϕ^4 periods*

We discuss the structure of ϕ^4 periods, focussing on the possibility that primitive ϕ^4 periods span a comodule for the motivic coaction. This is joint work with Oliver Schnetz and rests on a recently updated database of hundreds of exact results for primitive graphs with up to eleven loops.

SINAN ÜNVER – *Iterated sum series and p-adic multiple zeta values*

p-adic multi-zeta values are the p-adic periods of the unipotent fundamental group of the thrice punctured line. They turn out to give all the p-adic periods of mixed Tate motives over Z. In this talk, I will give an explicit series representation of these values in all depths. The new tool is a certain regularization trick for p-adic series.